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Gabriel Cuadra
Banco de México

Horacio Sapriza
Rutgers University

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Sovereign Default, Terms of Trade and Interest Rates in Emerging Markets*

Gabriel Cuadra†
Banco de México

Horacio Sapriza‡
Rutgers University

Abstract

Emerging economies tend to experience larger fluctuations in their terms of trade, countercyclical interest rates and more default episodes than developed countries. These structural features might suggest a relevant role for world prices in driving country spreads. This paper studies the role of terms of trade shocks in inducing output fluctuations and countercyclical spreads using a stochastic dynamic general equilibrium model of a small open economy. The model predicts that default incentives and default premia are higher in recessions, as observed in the data. In a quantitative exercise, the model matches various features of emerging economies and can account for the dynamics of default episodes in these markets.
Keywords: Default, Terms of Trade, Sovereign Debt.
JEL Classification: F34,F41

Resumen

Las economías emergentes tienden a experimentar mayores fluctuaciones en sus términos de intercambio y más episodios de default soberano que los países desarrollados. Estas características estructurales sugieren un papel relevante para los choques externos en la generación de fluctuaciones económicas y spreads. Este artículo estudia el papel de choques en los términos de intercambio sobre los incentivos a hacer default y las primas de riesgo, usando un modelo de equilibrio general para una economía pequeña y abierta. El modelo predice que los incentivos a no pagar la deuda son mayores en recesiones, como se observa en los datos. En el análisis cuantitativo, el modelo replica varias características de las economías emergentes.
Palabras Clave: Default, Términos de Intercambio, Deuda Soberana.

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† Dirección General de Investigación Económica. Email: gcuadra@banxico.org.mx.
‡ Rutgers Business School. Email: hsapriza@andromeda.rutgers.edu.

1. Introduction

Small open emerging market economies differ from developed economies in different aspects: developing countries typically face large swings in the price of the goods they export. These countries rely heavily on a narrow range of primary commodities for their export earnings, and are highly dependent on imported capital goods and intermediate inputs for domestic production. They also have more volatile business cycles, and are more crisis prone than developed countries. In the last two decades, emerging markets have experienced several cases of sovereign default, some of the most recent being Ecuador in 1999 and Argentina in 2001 among others. Additionally, these economies have countercyclical interest rates, which are in general related to countercyclical default risk. These structural features suggest that fluctuations in world prices may play a significant role on business cycle fluctuations and country interest rate spreads in developing economies.

The objective of this paper is to examine how the level and volatility of terms of trade affect incentives to default and thus, equilibrium interest rates in developing countries using a stochastic dynamic model of a small open economy with endogenous default risk. The economy produces an exportable good using labor and foreign inputs. Households value leisure and consume foreign and domestic goods and the domestic government has access to international financial markets, where it can borrow or lend to foreign lenders. Markets are incomplete because the government buys and sells one period non contingent discount bond and it cannot commit to repay its sovereign debt.

In a quantitative analysis, the model is calibrated to emerging economies. The paper shows that terms of trade have significant effects on country spreads through their impact on output and household consumption. The model generates countercyclical spreads as observed in data, explains a portion of the interest rate spread level and volatility and mimics the negative correlation between GDP and spreads to a significant extent. Results match the empirical data in that default incentives are higher when the economy is in recession, has large debt positions and terms of trade are relatively low. In addition, a more volatile output is associated with higher default probabilities and interest rate spreads.

The production technology in the paper allows one to explore the role of input shares for quantifiying country spreads in these economies. The model provides guidance on how would changes in the production structure of these economies impact on the countries' risk of default, and thus, on spreads. In particular, it is observed that a higher dependence of the productive sector on foreign inputs may lead to higher country interest rate spreads and lower debt levels.

In a similar way, by analyzing changes in the consumption structure between importables and exportables, the model shows that for any given level of output fluctuations, the higher the weight of imported goods in the household consumption basket, the higher and more volatile will be the spreads and the lower will be the sustainable debt level.

The paper proceeds as follows: Section 2 provides the link to the literature; the economic environment and the theoretical model are presented in Section 3, the equilibrium is characterized in Section 4, the quantitative implications of the model are analyzed in Section 5 and the conclusions are presented in Section 6. The algorithm is described in the appendix.

2. Link to the Literature

During the 1970s international lending took mostly the form of bank lending. The defaults of the 1980s prompted the international capital markets to partly switch from bank loans to bonds. In the 1990s, international bond issues by emerging market economies have surged dramatically and became one of the fastest growing devices of external development finance. An increasing number of emerging economies currently borrow from the international capital market by issuing bonds. Latin America accounted for the largest share of emerging market tradable debt throughout the decade. Bond financing became the second major source of funding in that region, where Cline (1995) documents that bond issues grew from less than 1 billion US$ in 1989 to 11.17 billion US$ in 1993. At the end of 1999, Latin America's tradable sovereign external debt stood beyond US$ 200 billion. (CEPAL 2000).

Interest rate spreads (the differences between yields on sovereign bonds of emerging market economies and U.S. treasury securities of comparable maturities) on bonds issued by Latin American countries fluctuated significantly during the decade and were subject to abrupt changes in moments of crisis. In fact, the change in composition of international capital lending has been accompanied by the occurrence of sudden reversals of capital flows and defaults like the Ecuadorian crisis of 1999 and the Argentinian crisis of 2001. Therefore, in order to assess the role of bonds as a source of external finance in emerging markets, it is important to understand the behavior and evolution of bond spreads in a framework that explicitly considers the link between bond value and risk of default, and the determinants of this risk.

There is an extensive debate about the determinants of pricing and thus the yield spreads of emerging market bond issues: while some subscribe to the view that spreads are driven by improvements in economic fundamentals, others attribute the changes in spreads to global factors such as international interest rates and market sentiment. The identification of the sources of income and country spread volatility is very relevant from an analytical as well as from a policy perspective, since it helps policy makers to address them. A number of studies have addressed this issue: several studies find that macroeconomic variables such as GDP growth and external debt to GDP ratios are important determinants of country spreads.[1] Min (1998) and Min , et. al. (2003) find that improved terms of trade are associated with lower yield spreads, since an improvement in the terms of trade implies an increase in exports earnings and better repayment capacity, which reduce the yield spread.

Terms of trade and commodity price volatility are an important factor in driving business cycle fluctuations and a potentially relevant source of interest rate spreads in emerging economies. These economies have more volatile business cycles than developed countries since their economies tend to be less diversified: GDP and government revenues sometimes depend excessively on one or two economic sectors or commodities. According to UNCTAD, in 1995, 57 developing countries depended on three commodities for more than half of their exports. The notorious volatility of commodity prices creates instability and uncertainty for commodity-dependent developing countries, undermining economic growth and affecting governments, consumers and producers.[2]

The literature on small open economies recognizes the terms of trade shocks as one of the most relevant shocks affecting these economies. Mendoza (1995) was the first to analyze the quantitative importance of terms of trade shocks in driving business cycles

[1] Goldman Sachs (2000) among others

[2] World Bank Premnote No.13

using a dynamic stochastic small open economy model. His work focuses on aggregate output fluctuations and he finds that terms of trade disturbances explain 56% of output in developing countries. Kose (2002) studies the role of world price shocks - fluctuations in the prices of capital, intermediate and primary goods, and in the world interest rate - in the generation and propagation of business cycles in small open developing countries. He finds that world price shocks play an important role in driving business cycles in small open emerging economies: roughly 88 percent of aggregate output fluctuations can be explained by these shocks.

Figures 1 to 4 in appendix illustrate the correlations between terms of trade fluctuations, interest rates spreads and GDP movements for Ecuador, the first country to default on Brady bonds, and Venezuela[3]. Before dollarizing the economy in January 2000, in 1999 Ecuador defaulted on its debt after a significant deterioration of the macroeconomic conditions. Prior to the crisis, the country experimented a significant deterioration of its terms of trade. Falling oil and primary commodity prices and a low nonoil tax base played a crucial role, which indicates the relevance of commodity price shocks.[4] Ecuador and Venezuela, and specially their exports are highly dependent on a few commodities.[5] Moreover, as figures 1 and 2 show, there is a strong positive correlation between one year lagged terms of trade movements and GDP fluctuations. This evidence is consistent with the findings in Broda (2003) who observes that terms of trade shocks are a substantial source of economic fluctuations and can seriously disrupt output growth in developing economies, specially in countries with fixed exchange rate

[3] GDP and TOT are in logs and all variables are detrended with a H-P filter.

[4] World Bank Report on Ecuador (May 2003) and S. Fischer, IMF (2000)

[5] Oil and bananas together accounted for 59 percent of Ecuadorian exports in 2001. Oil represented 70 percent of the exports of Venezuela in 1998.

regimes, and with the empirical study for Argentina and Ecuador in Broda and Tille (2003).

Recent empirical studies by Caballero (2003), Caballero and Panageas (2003) and by Calvo, Izquierdo and Mejia (2004) also find that a fall in commodity prices increases the likelihood of a Sudden Stop in capital flows, an event which is accompanied by large interest rate upswings. They point out the recent episode of the Russian crises of August 1998, in which emerging market economies such as Argentina, Chile, Colombia, Ecuador, Korea and Peru all suffered from sudden stops in capital flows and an increase in country risk premia. Table 1 and figure 5 show a strong positive comovement in terms of trade for many latin american countries over the last decade. This may suggest that terms of trade movements in the region might play a role in explaining the positive comovement of interest rate spreads, a feature claimed in some of these empirical studies.

The fact that sovereign spreads are higher in recessions indicates that business cycles in emerging market economies are negatively correlated with the interest rates that these economies face in international credit markets. In recent studies with data for different emerging economies, Neumeyer and Perri (2001) and Uribe and Yue (2004) have documented that the cost of foreign credit is higher in recessions than in booms. Emerging economies also present more volatile interest rates than developed countries. The countercyclical behavior of interest rates and their higher volatility are closely linked to the default probabilities that foreign lenders perceive from these economies. Empirical studies have estimated these probabilities, finding that the incentives to default are higher in recessions.[6] Therefore, we can consider that in a recession, an emerging

[6] See Marcel Peter (2002) for a survey on econometric studies of the probability of sovereign default in emerging markets.

economy faces higher interest rates precisely because in that phase of the cycle foreign lenders perceive higher default probabilities and thus require higher risk premia.

In addition, Cantor and Packer (1996) have found that higher sovereign credit ratings are associated to lower interest rates. They have also observed that these ratings, which are valuations on the probability that a borrower will not pay back its debts, strongly respond to macroeconomic factors, such as the GDP growth rate and per capita income. These facts also suggest that the countercyclical behavior of sovereign default risk may explain the countercyclicality of interest rates. Therefore, the probability of no repayment that is perceived by international creditors may be playing a relevant role in explaining the behavior of interest rates faced by emerging market economies. Figures 3 and 4 show the counter-cyclicality of interest rates spreads for Ecuador and Venezuela. This evidence suggests that the analysis of international credit markets and the joint study of default risk, interest rates and external price shocks are a relevant research topic for emerging market macroeconomics.

In standard general equilibrium models for small open economies, Mendoza (1991), it is assumed that there is perfect access to international credit markets, so the countries will borrow at a given international risk free rate up to the amount that their wealth allows them to. These countries participate in international credit markets in order to smooth their consumption and insure themselves against adverse shocks, facing no restriction in accessing the international credit markets. Because the international interest rate is exogenous, that is, it is not determined within the model, these models cannot explain the behavior of the interest rates that are faced by emerging market economies: interest rates do not depend on the domestic decisions taken by these economies.

With the objective of generating endogenous interest rates that vary with the business cycle, this paper introduces a credit friction in a stochastic dynamic model for a small open economy. The friction arises because the government has the option of defaulting on its foreign debt. Thus, the interest rate faced by the country results from the interaction between the government and the foreign creditors.

This paper considers a credit friction based on the willingness to pay of the borrower, who will optimally decide to default on her debt when the expected discounted value of defaulting exceeds the expected discounted value of repaying. Arellano and Mendoza (2002) discuss how this credit friction can be used to explain emerging markets features. The willingness to pay models were originated in the pioneering work of Eaton and Gerzovitz (1981) and using this approach, Chatterjee, Corbae, Nakajima and Rios Rull (2002) develop a quantitative model of unsecured consumer credit with the risk of default where they model equilibrium default in an incomplete markets setting.

This paper extends the approach developed by Eaton and Gersovitz (1981) in their seminal research on international lending by analyzing the relationship between terms of trade fluctuations and endogenous default probabilities in a production economy. The model considers a default penalty that is limited to temporary financial autarky and introduces a production technology. The model is applied in a quantitative exercise to a set of emerging market economies.

The model in this paper is specially related to Arellano (2003), who develops a quantitative model based on the willingness to pay approach in order to study output, real exchange rates and country spreads in emerging markets. Her analysis, however, does not consider a production economy or intermediate foreign inputs and does not address the issue of terms of trade fluctuations. Aguiar and Gopinath (2004) analyze the effect of stochastic productivity trends to improve the empirical predictions of the model on the countercyclicality of the current account. Yue(2004) develops a quantitative small open economy model to study sovereign default considering a debt renegotiation process.

The importance of terms of trade in a stochastic general equilibrium model of a small open economy is highlighted by Mendoza (1995) but he does not model endogenous equilibrium default. Kose (2002), Broda (2003) and Broda and Tille (2003) also study the importance of terms of trade and economic fluctuations using various approaches but their models do not account either for endogenous default risk premia or interest rate spreads.

3. The Model

Consider a standard neoclassical small open economy model with four agents, households, firms, government and foreign lenders, where the only asset traded in international financial markets is a one period non contingent real discount bond that is available to the government[7]. Debt contracts are not enforceable as the government has the option to default on them. When it defaults, it is temporarily cut off from credit markets. Foreign lenders charge a premium to account for the probability of not being paid back by the government. Households consume exportable and importable goods and leisure, and the government smooths their consumption path through lump sum transfers. Firms produce tradable goods using labor and foreign inputs.

The model economy in the paper is subject to terms of trade shocks P_t with a stationary and monotone Markov transition function. Let $Q_{s_{it}}(P_{t+1}|P_t)$ denote the Markov transition function for P.

[7] Most external debt in developing countries represents government debt. For example in Argentina and Ecuador, government debt accounts for almost 70% of the total stock of foreign debt.

3.1. Households

There is a representative agent with preferences given by the present value of the sum of instantaneous utility functions. She maximizes the following objective function:

$$\max_{\{x_t, m_t, l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C(x_t, m_t, l_t)) \tag{3.1}$$

where $\beta \in (0,1)$ is the discount factor. Households derive utility from the consumption of goods and leisure. Let C_t be a composite good that aggregates household consumption of importables, exportables and leisure. Thus, x_t represents the domestic consumption of the exportable good produced in this economy and m_t represents the amount of the importable good . The amount of labor is denoted by l_t. The period utility function is concave, strictly increasing and twice differentiable and follows the GHH (1988) specification:

$$u(C_t) = \frac{(c_t - \frac{l_t^{1+\Psi}}{1+\Psi})^{1-\sigma} - 1}{1-\sigma}$$

where σ represents the coefficient of risk aversion, Ψ determines the elasticity of labor supply and

$$c_t(x_t, m_t) = x_t^{\alpha} m_t^{1-\alpha}$$

is a composite good with a Cobb Douglas aggregation function on household consumption of importables and exportables.

The representative consumer receives labor income and a lump sum transfer from the government, so she maximizes 3.1 subject to the budget constraint

$$Px_t + m_t = w_t l_t + T_t$$

where P_t is the relative price of exportables with respect to importables, the terms of trade faced by the economy. Terms of trade are exogenous because the economy is small in international goods markets. w_t represents the wage, and T_t reflects a lump sum transfer in units of importables that the government gives to the household to smooth her consumption path. A positive T_t is thus a subsidy and a negative value represents a tax. This transfer is chosen by the government, so it is exogenous for individual consumers. Given the nature of the representative household's budget constraint, she faces a static optimization problem.

3.2. Firms

Firms are identical, perfectly competitive in input and final goods markets and produce domestic goods using labour L and a foreign input Z. They produce using a CES technology. Firms hire labor and buy an intermediate foreign good to produce an exportable final good.

The problem of the representative firm can be expressed as follows:

$$\max_{\{L_t, Z_t\}} P_t F(L_t, Z_t) - [w_t L_t + Z_t] \tag{3.2}$$

where $F(\cdot)$ is strictly increasing in each argument and homogeneous of degree one, L_t and Z_t denote respectively the amount of labor and imported intermediate good. While countries are in temporary financial autarky after a default episode, Puhan and Sturzenegger (2003) estimate an average fall in output of 2 % that may be reflecting cuts in foreign aid and trade disruptions and Rose (2003) finds an 8% decline in bilateral trade. The output loss in autarky is captured in the model as an exogenous fall in the productivity of firms, $AP_t F(L_t, Z_t)$ where $0 < A < 1$.

3.3. Government

There is a benevolent government in that its objective is to maximize the utility of the households in the economy. The government has access to international financial markets where it can lend and borrow. Credit markets are incomplete because the government can only save and indebt itself issuing and buying non contingent one period bonds. The government uses international borrowing to give a lump sum transfer T to the households (if $T < 0$ it is a tax). Each period, the government can choose between paying its debt or defaulting on it. This decision results from comparing the net benefits of the two alternatives, that is, by optimally balancing the cost of exclusion given by the foregone benefits of consumption smoothing, against the direct costs of repayment given by the short-run disutility of repaying the loan. Since the government faces an intertemporal problem, it is expressed in a recursive dynamic programming form. Conditional on having access to international credit markets, the government has to decide whether to default, the amount of transfers and how much borrowing or saving to do each period given the exogenous shocks and the amount of outstanding assets or liabilities it has. Therefore, the state variables for the government are B_t, d_t and P_t, where $d_t = 1$ if the economy has access to credit markets and 0 otherwise.

The value function of the government who has access to credit markets and begins the period with an amount of foreign assets B and shock P is $V_0(B, P)$. The government must decide whether to default or not by comparing the value of paying its debt and remaining in the credit market $V^c(B, P)$, with the value of defaulting and living in temporary autarky $V^d(P)$. The decision problem can be expressed as follows:

$$V_0(B, P) = \max\{V^c(B, P), V^d(P)\} \tag{3.3}$$

so that the optimal default decision can be characterized by

$$D(B,P)=\begin{cases} 1 & if \;\; V^{c}(B,P)>V^{d}(P) \\ 0 & otherwise \end{cases}$$

which indicates that the government optimally defaults whenever the discounted value of choosing to default is equal or higher than the continuation value. The government default policies determine a repayment set $\Gamma(B)$ defined as the set of values of the exogenous shocks such that repayment is optimal given asset holding level B,

$$\Gamma(B)=\{P\in\Upsilon : D(B,P)=1\}$$

and a default set $\digamma(B)$ defined as the set of values of the exogenous shocks such that default is optimal given asset holding level B,

$$\digamma(B)=\{P\in\Upsilon : D(B,P)=0\}$$

When the government decides to pay, it can issue new debt and give a transfer to the households according to the following restriction:

$$T=B-q(B',P)B'$$

A negative value of B implies that the government has foreign debt. $q(B',P)$ is the price of the bond that pays one unit of importables next period if the government does not default. When the government borrows, it sells bonds in the international credit market, and when it lends, it buys bonds from foreign creditors. A sell of B' in bonds - a negative value of B'- implies that the government receives $q(B',P)B'$ units of period t importable good from foreign creditors on the current period and promises to pay B' units of the good next period conditional on not defaulting.

In the same way, a purchase of bonds of value B' implies that the government lends $q(B', P)B'$ units of period t importable good to foreign creditors and it will receive B' units next period. It is assumed that foreign creditors always pay their debts, so the only agent who may decide not to commit to repay is the domestic government. When the government borrows, the price of the bond reflects the possibility that the government defaults, so this price should depend on B' (the amount that is borrowed) and on P (since today's shock affects the probability distribution for next period's shock) because the incentives to default depend on both factors.

Therefore, the government problem when it participates in international credit markets can be expressed as follows:

$$
\begin{aligned}
V^c(B,P) &= \max_{T,B'}\{u(C\,(X,M,L)) + \beta \sum_{P'} V_0(B',P')Q(P'/P)\} \qquad (3.4) \\
&\quad s.t. \\
B &= q(B',P)B' + T \\
M &= M(T,P) \\
X &= X(T,P) \\
L &= L(T,P)
\end{aligned}
$$

The first restriction is the one already described. The last three equations reflect the fact that the government makes its decisions subject to the private sector optimizing as well. X, M and L represents the optimal consumption and labor decision rules when the country has access to credit markets d=1. Note that households optimize taking as given the government transfer T.

When the government decides not to pay its sovereign debt the country loses access to international credit markets for a stochastic number of periods, so the country is temporarily in financial autarky, without being able to save or borrow. Therefore, the problem when the country is in autarky is as follows:

$$V^d(P) = u(C_d\left(X_d, M_d, L_d\right)) + \beta \sum_{P'} [\mu V_0(0, P') + (1-\mu) V^d(P')] Q(P'/P) \tag{3.5}$$

$$\begin{aligned} & s.t. \\ T_d &= 0 \\ X_d &= X_d(T_d, P) \\ M_d &= M_d(T_d, P) \\ L_d &= L_d(T_d, P) \end{aligned}$$

which shows that the government has no instruments now to smooth household consumption. X_d, M_d and L_d represents the optimal consumption and labor decision rules when the country is in autarky d=0. The probability of reentering financial markets next period is denoted μ. When the economy returns to financial markets, it does so with no debt burden, $B = 0$.

3.4. Foreign Creditors

There is a large number of identical, infinitely lived foreign lenders. Each lender can borrow or lend resources at the risk free rate r_f and participates in a perfectly competitive market to the small open economy.

The individual lender is risk neutral and maximizes expected profits, which are given by the following equation

$$\Phi = -qB' + \frac{\lambda(B', P)}{1 + r_f} 0 + \frac{(1 - \lambda(B', P))}{1 + r_f} B'$$

As it was pointed out by Cole and Kehoe (1996), the assumption of risk neutrality of lenders captures the idea that the analysis considers that compared to international credit markets, the domestic economy is small.

The first term of the equation above shows that when creditors lend to the government in the current period, they buy the discount bond issued by the domestic government at a price q. Next period the lenders may receive the face value of the bond depending on whether the government defaults or not. When it defaults, creditors get 0 units of the importable good. $\lambda(B', P)$ is the endogenous probability that the government defaults on its sovereign debt. Therefore, with probability $1 - \lambda(B', P)$ lenders will be paid back an amount B'

Since there is perfect competition in the credit market, a zero profit condition for the foreign creditor is satisfied. The bond price is then:

$$q(B', P) = \frac{(1 - \lambda(B', P))}{1 + r_f} \tag{3.6}$$

where $\lambda(B', P)$ is the endogenous probability that the domestic government will default on its sovereign debt. Thus, the equilibrium bond price $q(B', P)$ reflects the probability of default of the government, $\lambda(B', P)$, which results from

$$\lambda(B', P) = \sum_{P' \in F(B')} Q(P'/P)$$

so that the default probability is zero when $F(B') = \varnothing$ and it is one when $F(B') = \Upsilon$.

4. Equilibrium

In equilibrium households choose optimal consumption of importables and exportables given prices and government transfers, firms optimize given prices and the government determines its optimal default policy and its optimal asset holding policy subject to the private sector optimizing and foreign lenders optimizing by satisfying their zero profit condition from the debt contract.

Definition 4.1. *A recursive equilibrium for this small open economy is characterized by*

i. *a set of value functions* V_0, V^c *and* V^d *for the government,*

ii. *a set of policy functions for household's consumption of exportables* X, X_d *importables* M, M_d *and labor supply* l, l_d,

iii. *policy functions for the firm's demand for foreign inputs* Z,Z_d *and for labor* L,L_d,

iv. *policy functions for government's default decision* D, *optimal asset holdings* B' *and transfers* T, T_d,

v. *wage functions* w, w_d

vi. *a bond price function* q

such that

1. *The wage function is such that the labor market clears,* $L = l$ *when the country is in trade and* $L_d = l_d$ *when the country is in autarky.*

2. *Given the wage function, the policy functions for the firm solve 3.2:*

$$\{Z, L\} \in \arg\max_{\{L,Z\}} PF(L, Z) - [wL + Z]$$

3. *Given the government policies, the bond price function and the wage function, the household policies for consumption and labor solve the household's problem 3.1.*

4. *Given the bond price function* q *and the optimal policies for firms and households, the government's value functions* V_0, V^c *and* V^d *and its policy functions* D, B' *and* T *solve 3.3, 3.4 and 3.5:*

5. The equilibrium bond price $q(B', s)$ is such that all agents in the small open economy are optimizing and international lenders get zero expected profits, satisfying credit market clearing condition

4.1. Characterization of the Equilibrium

Proposition 1 *For any given realization of the exogenous shock P, default incentives are stronger the higher the level of foreign debt.*

Proof. See Appendix. $\blacksquare$

The logic is similar to Eaton and Gersovitz (1981). If default is optimal for some level of assets B for a given terms of trade shock P, the value of staying in the contract is lower than the value of default. Since the value of default is independent of B and the value of the contract monotonically increases in the amount of assets B, there exists a sufficiently negative level of assets $\underline{B}$ for which the economy optimally defaults for all realizations of the terms of trade shocks. As assets increase, the default set shrinks. At a sufficiently high level of assets $\overline{B} \leq 0$ the economy never defaults.

This proposition shows how the bond price schedule depends on the amount of assets and shock realizations. For $B \geq \overline{B}$ and any realization of the shock, default is not an outcome, so the bond pays the risk free rate. Since default sets shrink in assets, as the economy decreases its level of assets B'such that $\underline{B} < B' < \overline{B}$, default probabilities increase and the equilibrium bond price function $q(B', P)$ declines in B'. Therefore, the larger the amount of debt, the higher the default probability and the lower the bond price. When $B' \leq \underline{B}$ default is optimal for any realization of the shocks, which implies that the probability of default is one and thus the bond price is zero.

Figure 7 shows the default region for the calibrated economy, i.e., the combinations of terms of trade and foreign debt levels for which default is optimal. Given a terms of trade shock, if default is optimal for a certain debt level, it will be optimal for all higher levels of the ratio, illustrating the result of Proposition 1.

Proposition 2 *For any given level of foreign debt, without government intervention a lower realization of the exogenous shock P reduces household welfare.*

Proof. See Appendix. ■

An adverse terms of trade shocks has two effects on the economy: on the production side, as the relative price of output with respect to factor prices declines, the economy finds it more costly to produce the final exportable good, so the demand for labor and the foreign input falls. The decrease in the use of the foreign input lowers the marginal product of labor, thus decreasing further the demand for this factor and decreasing wages. Then, household income falls both because the economy produces a smaller amount of exportables and because the value of those produced goods can be exchanged for a reduced amount of imported goods. On the consumption side, adverse terms of trade change the relative price of the components of the consumption basket, making importables more expensive, which tends to reduce the consumption of importables. At the same time, the substitution effect tends to increase consumption of exportables. However, given that both importables and exportables are normal goods , the fall in income tends to reduce the consumption of both goods. Since the latter effect is stronger, consumption of both goods falls. The decline in labor implies a higher amount of leisure which increases welfare, but it also implies a lower household income that tends to reduce consumption of goods which more than offsets the leisure effect. The overall net effect is a fall in consumption of the composite good and therefore in household welfare.

The following propositions are derived for the case of i.i.d. shocks, permanent autarky and no output loss in autarky, but they are also satisfied in the specification for the quantitative analysis.

Proposition 3 *For any given level of foreign debt, default incentives are stronger the lower the realization of the exogenous shock P.*

Proof. See Appendix. ■

According to Proposition 2, an adverse terms of trade shock would reduce household's welfare, providing an incentive to the benevolent government in the economy to implement a positive transfer. However, if the government is a net debtor it would have to borrow in order to obtain resources for both paying back its foreign debt and giving a transfer to consumers, necessarily increasing foreign debt. After a sequence of bad terms of trade shocks the government could end up highly indebted. Under this circumstance, there would not be financial contracts available to the government that allowed it to give a positive transfer, so instead of implementing a transfer the government would have to tax households to pay back its debt. This would imply a lower level of welfare for consumers and since utility is concave and increasing in the composite good, the reduction in welfare would be more costly the lower the terms of trade. Therefore, if the government is already highly indebted and the country faces a negative shock, the bond is not a very useful insurance instrument because with a low terms of trade shock the economy would like to borrow but it cannot raise enough resources to smooth consumption. Then, the asset is not very valuable for the economy and default may be the optimal choice in a state where the shock is low and defaulting could be the optimal decision. Then, given a foreign debt level, the worse the terms of trade shock the higher the incentives to default.

As it can be seen in Figure 1 for the calibrated economy, given a debt level, if default is optimal for a certain terms of trade shock, it will be optimal for all lower values of the shock, implying that default incentives are stronger the lower the realization of the shock.

Proposition 4 *Asset decisions are increasing in terms of trade. The government borrows more in recessions than in booms. Given a current level of foreign assets* B*, conditional on not defaulting, for all* $P_1 < P_2$*, if* $B_1' = B'(B, P_1)$ *and* $B_2' = B'(B, P_2)$ *then* $B_1' < B_2'$*.*

Proof. See Appendix. ■

Since the government is benevolent, it maximizes consumer lifetime utility, which implies smoothing households's consumption. In order to do that, the government borrows and implements a positive transfer in bad times and taxes households and saves in good times. The government lends and borrows by issuing and buying non contingent bonds respectively. Since the government cannot commit to pay its foreign debt, when borrowing it faces an interest rate that may be higher than the risk free international interest rate. This risk premia is increasing in foreign debt because the incentives to default are higher when the government is highly indebted. Therefore, the bond price is increasing in foreign assets. On the other hand, when terms of trade shocks are i.i.d., bond prices do not depend on the current realization of terms of trade, because the probability of having an adverse shock tomorrow does not depend on the current level of the shock. Thus the government faces the same bond price schedule in all states of the world. The financial contracts available to the government are the same for all possible realizations of terms of trade. Now suppose that the government borrows more and gives a higher transfer to consumers in booms than in recessions: that would make household consumption even more volatile. Instead of smoothing consumption, the government would be doing the opposite. Since the contracts available to the government in good times are also available in bad times, the government could switch assets decisions and in this way smooth households consumption, which would be welfare enhancing for consumers, because they prefer a smooth path of consumption over a volatile one. Therefore, given the contracts available to the government, borrowing more in economic expansions would not be an optimal decision.

Proposition 5 *Interest rates cannot be procyclical. Given a current level of foreign assets B, for all $P_1 < P_2$, if $B_1' = B'(B, P_1)$ and $B_2' = B'(B, P_2)$ then $q(B_1') \leqslant q(B_2')$.*

Proof. See Appendix. ■

Governments borrow more in bad times at higher interest rates. When the country is hit by an adverse shock, the government issues bonds and transfers resources to consumers in order to avoid a fall in households welfare. Higher borrowing implies higher risk premium, because default incentives are stronger the higher the level of debt. Since adverse terms of trade realizations reduce domestic output, the government would be borrowing more in recessions and paying higher interest rates. Given i.i.d. terms of trade shocks, the government faces the same bond price schedule in good and bad times. The bond price is decreasing in foreign debt because risk premium is increasing in the probability of default, hence higher borrowing implies a movement along a unique and decreasing bond price schedule. Thus, interest rates are countercyclical.

4.2. Euler Equation

The Euler equation for the government is obtained from the first order condition of its dynamic optimization problem and the envelope theorem:

$$\frac{\partial u\left(C\right)}{\partial C}\left[\frac{\partial C}{\partial X}\frac{\partial X}{\partial T}+\frac{\partial C}{\partial M}\frac{\partial M}{\partial T}\right]\left[q+\frac{\partial q}{\partial B'}B'\right]=\beta\sum_{F(B')}\frac{\partial u\left(C'\right)}{\partial C'}\left[\frac{\partial C'}{\partial X'}\frac{\partial X'}{\partial T'}+\frac{\partial C'}{\partial M'}\frac{\partial M'}{\partial T'}\right]Q(P'/P).$$

The Euler equation is interpreted in terms of marginal benefits and marginal cost of additional lending or borrowing. First, consider the case where the government is a net creditor. Note that lending implies buying foreign bonds, so additional lending would imply additional bond purchases: the government would tax households and use the proceeds to finance these additional foreign bonds purchases. Since the country is small in international credit markets, all external prices including the world interest rate are given, so the government can not influence credit markets and additional lending would not affect the international risk free rate $\frac{\partial q}{\partial B'}=0$. Thus, the price of the bonds bought by the government is simply the inverse of the risk rate $q=\frac{1}{1+r_f}$, so that for each unit of additional lending the government increases taxes and reduces current households's expenditure in q units. In the next period, the government would receive one unit of foreign goods, which will be given to households as a lump sum transfer.

The marginal benefits and costs can be described as follows:

In terms of effects on today's utility, a marginal increase in current lending affects consumption of both goods. The government increases taxes and reduces consumer's disposable income in q units. Since both goods are normal, households end up consuming less of both exportable and importable goods, which would reduce today's welfare. On the other hand, the following period the government will get one unit of foreign goods for each unit of additional lending. These resources will be given to households as a lump sum transfer, thus future disposable income goes up, which allows households to consume more goods, increasing tomorrow's welfare.

Now consider the more interesting case where the government is a net debtor. Borrowing from international capital markets implies issuing bonds and transferring resources to consumers. Additional borrowing would affect government's transfers to households. Since the government is borrowing more it could give a higher transfer but at the same time if the government is already indebted additional borrowing would increase the interest rate that it faces in international markets. As it was explained before, default probabilities are increasing in foreign debt, so if the government wants to borrow more, foreign lenders would require a higher risk premium. The decrease in the bond price implies that the government is not getting as many resources from selling bonds as it would get if the bond price were constant. Therefore, when evaluating the benefits and costs of additional external borrowing, the government takes into account the fact that issuing more bonds would decrease the price. Although the country is small, the government's borrowing can affect the idiosyncratic bond price that it faces: if the government issues additional bonds, default probabilities will be higher and foreign creditors will demand a lower price, otherwise they will not buy the bonds.

In terms of effects on current welfare, for each unit of additional borrowing the government increases the transfer to consumers in $q + \frac{\partial q}{\partial B'} B'$ units, these extra resources allow households to consume more of both goods, thus today's utility goes up. Next period, the government would have to repay its debt, which reduces future household disposable income and consumption. However, it only pays back when it's optimal, so households only end up consuming less goods in those states where the government does not default and taxes households in order to honor its obligations. It is important to mention that the repayment set $F(B')$, depends on B', thus additional borrowing would enlarge it. In order to analyze the effects of extra borrowing on next period' utility, tomorrow's terms of trade realizations can be separated into three sets: first, those states where the government repays its debt, before and after increasing B'. In these states additional borrowing would reduce disposable income and consumption, since the government would have to tax households to pay its debt. Second, those states where the government defaults, before and after increasing B'. In these states, households simply consume their income, thus additional borrowing would not have any effect on welfare. Finally, those states where the government does not default before increasing B', but it does after issuing additional bonds. In these states, if households are taxed before the change in B', then they would not be taxed after the rise in external borrowing, therefore welfare would be increased in these states.

5. Calibration

5.1. Data

The benchmark model is calibrated to a set of emerging market economies considering quarterly data for the period 1994-2004. The data is obtained from different sources: terms of trade are constructed from IMF data and output are obtained from IMF. Spreads correspond to the Emerging Markets Bond Index Plus (EMBI+) constructed by J.P.Morgan[8]. Output and terms of trade are in logs and all series are filtered with the Hodrick-Prescott filter. The countries that were included in the sample are Argentina, Colombia, Ecuador, Peru and Venezuela.

Table1 in the appendix shows the terms of trade volatility for each country and the

[8] Regarding the EMBI spread the sample varies across countries: Argentina 1994:1-2004:4, Colombia 1999:3-2004:4, Ecuador 1995:2-2004:4, Peru 1997:2-2004:4 and Venezuela 1998:1-2004:4

cross country correlation. Venezuela stands as the country with highest terms of trade volatility, mainly due to its extremely high dependency on oil. Argentina, the country with the highest diversification in production (though still highly dependent on specific goods like beef and agricultural products), presents the lowest volatility in terms of trade. The cross country correlation values are all positive and average almost 0.4, with the largest being 0.85 between Ecuador and Venezuela. Figure 4 illustrates the positive comovement of the sample series. Table 1 shows the volatility in terms of trade, output and spreads, and the correlations between output and spreads for the different countries.

Table 3 depicts the lagged impact of terms of trade shocks on output for the five economies. Lags up to four quarters are considered. While the contemporaneous correlation is generally mild, lagged values of terms of trade present large values for most countries. The largest values correspond to Ecuador with 0.72 and a four quarters lag, while Peru is the country where the strongest correlation occurs with one quarter lag, and a value of 0.59.

5.2. The Benchmark Model

This calibration involved choosing the functional forms and the parameter values. The parameters are chosen based on existing empirical work on emerging markets, if available. Otherwise they are set to mimic some empirical regularities of emerging markets.

The period utility function has the GHH (1988) specification. This preference specification has the property that the marginal rate of substitution between consumption and labor is independent of consumption. Therefore, labor supply does not depend on the level of consumption. The composite consumption good c_t is obtained using a Cobb-Douglas aggregation function on the consumption of importables and exportables, where α represents the share of exportables. This specification of preferences generates the following optimal decisions:

$$\begin{aligned} m_t &= (1-\alpha)(w_t L_t + T_t) \\ x_t &= \frac{\alpha(w_t L_t + T_t)}{P_t} \\ L_t &= \left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{w_t}{P_t^{\alpha}}\right]^{\frac{1}{\Psi}} \end{aligned}$$

Firms in the benchmark economy produce the final exportable good with a Cobb Douglas production function,

$$F(L_t, Z_t) = L_t^{\theta} Z_t^{1-\theta}$$

a standard technology in dynamic general equilibrium models where $1-\theta$ corresponds to the intermediate good share. Thus, input demands follow from first order conditions for L and Z respectively:

$$\begin{aligned} P_t \theta L_t^{\theta-1} Z_t^{1-\theta} &= w_t \\ P_t(1-\theta) L_t^{\theta} Z_t^{-\theta} &= 1 \end{aligned}$$

Some of the parameter values that are used are standard for business cycles models in emerging markets. The parameter σ, the coefficient of relative risk aversion, is set equal to 2, a standard value. The parameter Ψ is set to 0.455 following Mendoza (1991). This parameter determines the labour supply elasticity, which equals $\frac{1}{\Psi}$.

The discount factor is set at 0.85 and helps to match the maximum risk free debt/GDP limit of 16%. According to Reinhart, Rogoff and Savasano (2003) safe external debt to GDP thresholds for emerging economies appear to be as low as 15 to 20 percent of GDP.

The share of exportables α is set to 0.85 which implies that consumption of final imported goods represents 15% of total expenditures on consumption goods. This value is taken from data of the Ecuador economy for the period under study, where the fraction of final imported goods in total household consumption fluctuated between 13 % and 16 %. The parameter μ reflects the exogenous probability of reentering international capital markets after default and it is set equal to 0.1. This value implies that a defaulting country will return to financial markets in about 10 quarters after defaulting on its foreign debt. This is in line with the exclusion period observed in the data by Gelos (2003), who calculated the average number of years that a country was excluded from international financial markets to be close to 3 years for countries that defaulted on their foreign debt during the period 1980 - 1999.

The parameter A governs the output loss in autarky and is set equal to 0.98 following empirical studies. Puhan and Sturzenegger (2003) estimate an average fall in output of 2% per year after a default for the default cases of 1980's. The foreign input share $1-\theta$ is set to 0.39 which is value used by Kose (2002). He finds an average value of 0.39 for this parameter using data for 28 developing countries. Shocks to terms of trade follow a first order autoregressive processes such that if $p_t = \log P_t$ then

$$p_{t+1} = \rho_p p_t + \varepsilon_{t+1}^p, \tag{5.1}$$

where ε_{t+1}^p is an IID standard normal shock. Emerging markets data is used in order to estimate the autoregressive process for the terms of trade shock. Once we have the parameters for the terms of trade shocks, we approximate them with a discrete first order Markov chain using Hussey and Tauchen's (1991) procedure. The parameters for the benchmark model are shown in Table 4.

5.3. Results

This section shows the main results of the paper, and analyzes the statistical properties of the model economy when it is subject to terms of trade shocks.

The business cycles moments for the benchmark calibration of the model are presented in Table 5. Business cycles statistics are average values over 100 simulations of 100 realizations each. The simulated data is log and filtered equally as the empirical data. Mean output is normalized to one. The model can match several features of emerging markets economies.

The correlation of output with spreads is consistent with the emerging markets data, though the magnitude is lower. The result that output is negatively correlated with spreads is due to the asset structure of the model. Asset markets are incomplete because there is only one asset available, a one period non-contingent bond. Given this market structure, default is tempting in times when output and consumption are low since a given debt-service payment reduces utility more strongly in those states. Since repayment of non-contingent loans are more costly in bad states of nature, incentives to default tend to be stronger in times of low output. Risk neutral creditors are willing to supply loans that in bad states of the world will result in default by charging a higher risk premium. In this way, the model can generate counter-cyclical interest rate spreads.

Figure 6 plots the discount bond price schedule as a function of assets for the highest and lowest values of terms of trade. As the figure shows, bond prices are an increasing function of foreign assets. For small levels of foreign debt, the government always pays back its debt, so it borrows from international markets at the world risk free interest rate. In this range of debt, bond price is simply the inverse of the gross risk free rate. For values of foreign debt up to 16.07 % of mean output, the government does not have any incentive to default so it still faces risk free interest rates. However, as foreign debt goes up, at a certain debt/GDP level bond prices start to decrease. This threshold level is increasing in the magnitude of the shock. The higher the levels of foreign debt the lower the bond prices because the incentives to default are stronger for large indebted governments. At debt levels above 20.29 % of mean output the government always defaults regardless of the terms of trade realization. At that point bond prices are zero since the probability of default is one.

For each of the terms of trade values, there is a range of borrowing levels for which default is a possible outcome, for these assets positions the bond price is between the inverse of the risk free rate and zero. For a given value of assets, the higher the terms of trade the higher the bond price. This implies that incentives to default are stronger when the economy suffers an adverse terms of trade shock. Therefore, the model predicts that default is more likely in bad times.

Figure 7 shows the default region for the calibrated economy, i.e., the combinations of terms of trade levels and foreign debt levels for which default is optimal. Given a terms of trade shock, if default is optimal for a certain debt level, it will be optimal for all higher levels of foreign debt.

Although default occurs in equilibrium, it is not a frequent event as it occurs on average only 32 times in 10,000 periods, implying that the country defaults every 78 years. This result is closely linked to the shape of the bond price schedule. As figure 6 shows, the schedule is extremely steep over the range of foreign debt levels that carry a positive and finite risk premia. Suppose that the current level of terms of trade is low, then this adverse shock lowers domestic output through its effects on imported inputs and reduces consumption possibilities since the purchasing power of household income is lower. Under this circumstances, the government would try to borrow from international markets in order to smooth private agents' consumption. As the government borrows, the bond price starts to decline, and at a certain debt level it decreases very sharply. At that point if the government wants to borrow any additional amount, it would have to pay a much higher interest rate, that is, the marginal cost of borrowing increases sharply. The government takes into account the effect of additional borrowing on the interest rate it has to pay and therefore decides not to go too far smoothing households consumption.

The soon and sudden interest rate increase means that bonds are not good instruments for insurance and consumption smoothing purposes anymore. So the government borrows from capital markets paying either low risk premia or not premia at all. Since low levels of risk premia are related to very low probabilities of default, it happens that default is a rare event. This feature also explains the high volatility of consumption obtained in the model: the ratio of $\frac{\sigma_c}{\sigma_{GDP}} \approx 0.98$ obtained in the simulation is clear evidence that the bonds are not good instruments for consumption smoothing.

Spread volatility is lower than the one observed in the data for Ecuador. The standard deviation of spreads obtained in the benchmark model is 0.69. The lack of spread volatility in the model derives from the fact that default is not a frequent event.

The simulated sequence plotted in Figure 8 shows how the interest rate spread increases as terms of trade deteriorate in the economy: from periods two to five the persistent deterioration in terms of trade leads to an increase of more than two percentage points in country spread, though on the next period the terms of trade recovers and the spread declines. However, the deeper fall in the terms of trade from periods 10 to 16 ends in increasing spreads and optimal default of the economy. As can be seen, terms of trade and interest rate spreads are negatively correlated and default occurs in bad times, as observed in data.

5.4. Sensitivity Analysis

This section studies how changes in the technological characteristics and in household's consumption basket in the benchmark model affect interest rate spreads, debt limits and default rates in the small open economy. As the participation of importables in household's consumption basket becomes more important, equilibrium spreads and default rates are higher and the range of debt for which there is a positive and finite risk premium becomes broader. The economy starts paying a premium at lower debt level but the debt limit for the economy increases. A similar effect is observed when the share of foreign inputs in output increases.

Table 6 reports the results of the sensitivity analysis, different values for the foreign input share and the domestic good share were considered.

As Table 6 shows, a change in the factor share of the foreign input affects equilibrium spreads and debt sustainability levels for a given terms of trade shock. The larger the share of foreign inputs in production, the more volatile the domestic output. Consider a deterioration in terms of trade, firms would try to produce less exportable goods, in order to do that they would hire less labor services and purchase less foreign inputs, these would reduce wage rates without affecting prices of foreign inputs. A lower wage rate would moderate the output fall, nevertheless this effect is less important the lower the share of labor in production. Thus for any given adverse terms of trade shock, a larger share of Z in output implies a deeper recession that would make households worse off, requiring larger government borrowing to smooth consumption.

There are two effects on the incentives to default: first, a larger share of foreign inputs implies a more volatile output, so in the absence of borrowing or lending the fluctuations in household consumption will be larger and because of that, having access to international credit markets will be more valuable. Because the value of participation is higher, the economy might tend to borrow more without having incentives to default. Second, under adverse shocks, recessions are more severe the larger the weight of foreign inputs, so repayment of non-contingent debt is more costly in those states, tending to increase default probabilities and interest rate spreads, i.e., the incentives for the small open economy to default start at lower debt levels.

Figure 9 illustrates that the risk free debt limit is lower, reflecting the fact that the second effect dominates in the model economy. In this way, a higher share of foreign inputs and a more volatile output would imply higher country spreads and lower sustainable debt levels. These results are consistent with data and empirical studies: Catao and Sutton (2002) who find that countries that experience higher terms of trade and GDP volatility encounter higher probabilities of default. Gelos and Shahay (2003) find that countries with higher terms of trade and output volatility have less access to international credit markets.

As output and overall consumption becomes more volatile, the shock is more important for this economy relative to the amount of debt, so the bond price schedule is less steep. It follows that the default region for this economy now becomes broader, as the figure shows. As mentioned before, for a given terms of trade shock, domestic output is more volatile the larger the share of foreign inputs. Thus a small share of Z implies small differences among output and consumption in good times and output and consumption in bad times. Since the difference in utility between a high shock and a low shock will be smaller for any given level of foreign debt, the amount of debt becomes a more important factor for the default decision, implying a steeper bond price schedule. Conversely, a higher participation of foreign inputs in production leads to a stronger relative effect of terms of trade shocks compared to debt, and therefore a flatter bond schedule and broader region with positive and finite risk premia.

It can also be observed from Table 6 that a given worsening in the terms of trade should induce a larger decline in welfare and larger spreads, the larger the share of importables in household consumption preferences. An adverse terms of trade shock has two effects: first, a higher price of imported inputs implies a lower output and lower labor income for households. In second place, the purchasing power of exportable goods produced by the country will be lower, so the value of labor income in terms of importables also decreases and the original set of consumption bundles is now unavailable to the representative household. Therefore, the higher the weight of importables in households' preferences, the more severe will be the latter effect since the good to which households are giving increasing value in its utility function becomes more expensive. Hence, for a given terms of trade shock and output loss, the negative effects on households' welfare will be stronger. As with the share of foreign inputs in output, a larger participation of importables in consumption makes participation in credit markets more valuable and thus more borrowing tends to be supported without having incentives to default. However, as mentioned before, an adverse terms of trade shock reduces both output and the purchasing power of each unit of domestic good. As the share of importables increases the second effect becomes more important, so that any given terms of trade shock has a stronger effect on household welfare. This implies that repayment in those states is more costly, thus default incentives tend to rise, the sustainable amount of debt reduces and default probabilities and spreads increase. As in the case of foreign inputs in production, simulation results show that the latter mechanism clearly dominates in the calibrated economy. Therefore, the overall effect of a higher share of importables is an increase in country spreads and a decrease in the sustainable debt level for a given output and labour income levels and volatility.

6. Conclusions

The paper analyzes the importance of terms of trade fluctuations on country spreads and default incentives in emerging markets by considering a stochastic general equilibrium model of a small open economy where default is an equilibrium outcome. Default probabilities are endogenous to the economy's incentives to default and they affect the equilibrium interest rates faced by the economy. Markets are incomplete, which generate counter-cyclical default risk since it is more costly to repay non-contingent loans in times when terms of trade are low and therefore output and consumption are low, than in booms.

In times where the government is highly indebted, a negative terms of trade shock may induce the government to default on its sovereign debt. A deterioration of terms of trade can trigger an economic contraction in the form of a decrease in the real value of output and therefore a decline in consumption. This can induce a highly indebted government to optimally decide not to raise taxes to repay the debt, which would exacerbate households' fall in consumption, but to default instead on its sovereign debt in order to avoid such decline in consumption. Through these shocks, the model generates counter-cyclical interest rate spreads, explains a significant portion of the spread level and volatility and mimics the correlations of GDP with spreads to a significant extent in the quantitative analysis.

The model also analyzes how would changes in the production structure of these economies impact on the countries' spreads. The production technology in the paper allows one to explore the effects of input shares on incentives to default and therefore on country spreads in these economies. In particular, it is observed that a higher dependence of the productive sector on foreign inputs may lead to higher country interest rate spreads and lower equilibrium debt levels. Similarly, when analyzing the effects of changes in the consumption structure between importables and exportables, the model predicts that for any given production structure, the higher the weight of imported goods in the household consumption basket, the higher and more volatile will be the spreads and the tighter the sustainable debt level. The larger the share of importables in consumption that households want to hold, the more severe is the fall in their overall consumption, since it has become more expensive and their income declined. In the same way, the larger the foreign input share in production, the worse the recession for a given adverse terms of trade realization, so repayment is more costly, providing higher incentives to default. Foreign creditors perceive this increased riskiness of the economy, thus lending at higher interest rates in equilibrium and reducing the amount of lending to the small economy for any given interest rate spread: higher interest rate spreads and lower debt levels are observed in equilibrium.

In light of the above results, it is possible to derive some interesting policy lessons: given the strong effects of terms of trade shocks on government incentives to default and thus on interest rate spreads, it seems important for many developing countries that governments provide incentives to the private sector to diversify the export structure and provide financial instruments for the private sector to smooth price fluctuations.

The latter might include the development of credit markets for commodity price indexed bonds, which would ensure, as efficiency dictates, that countries pay more in good states than in bad ones.

7. Appendix

7.1. Algorithm

From the optimal labor decision of the household and the first order conditions of the firm, the household labor income, wl, is calculated for each possible realization of the shocks. Given the equilibrium in the labor market, the expression for the labor supply is substituted in the first order conditions of the firm problem. In order to solve for Z, l and w for each of the states of the economy, the following system of nonlinear equations is solved:

$$\begin{aligned} P\theta\,(L)^{\theta-1}\,Z^{1-\theta} &= w \\ P(1-\theta)\,(L)^{\theta}\,Z^{-\theta} &= 1 \end{aligned}$$

$$L = l = [\alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{w_t}{P^{\alpha}}]^{\frac{1}{\Psi}}$$

Once the above system is solved for all possible values of the shocks, the following algorithm is used:

1. Assume an initial function for the price of the bond $q_0(B', P)$. To calculate the initial value of the bond, use the inverse of the risk free rate.

2. Use q_0 and the initial values of V_0 and V^d (eg. start with 0 matrices) to iterate on the Bellman equations and solve for the value functions and the policy functions.

3. Given the initial price of the bond q_0, calculate the default function $D_0(B,P)$, and then update the price of the bond using the following equation:

$$q_1 = \frac{(1 - D_0(B,P))}{1 + r_f}$$

4. Use the updated price of the bond q_1 to repeat steps 1, 2 and 3 until the following condition is satisfied:

$$q_i - \frac{(1 - D_i(B,P))}{1 + r_f} < \epsilon$$

where i represents the number of iterations and ϵ is a small number.

7.2. Propositions

Proposition 1 *For any given realization of the exogenous shock P, default incentives are stronger the higher the level of foreign debt B.*

Proof. For each realization of P, default incentives are stronger the lower the value of foreign assets. This is derived from the fact that V^c, the value of staying in good credit, is strictly increasing in foreign assets, while V^d, the value of defaulting, does not depend on the level of assets. Therefore, there is a unique value for assets $B^*(P)$ where $V^c = V^d$, such that given P the government always pays back if foreign assets are higher than $B^*(P)$ and defaults otherwise.

Thus, the only proof that is needed is that V^c is strictly increasing in foreign assets, $\frac{\partial V^c}{\partial B} > 0$. The envelope theorem is used on V^c to derive this result and the following expression is obtained:

$$\frac{\partial V^c}{\partial B} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial T}\frac{\partial T}{\partial B} + \frac{\partial u}{\partial m}\frac{\partial m}{\partial T}\frac{\partial T}{\partial B} + \frac{\partial u}{\partial l}\frac{\partial l}{\partial T}\frac{\partial T}{\partial B}.$$

Given the GHH specification for household utility, labor supply does not depend on the level of consumption, $l = [\alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{w}{P^{\alpha}}]^{\frac{1}{\Psi}}$, thus $\frac{\partial l}{\partial T} = 0$. In addition, the transfer that households receive from the government, $T = B - q(B', s)B'$ is increasing in the level of foreign assets, $\frac{\partial T}{\partial B} = 1$ so that

$$\frac{\partial V^c}{\partial B} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial T} + \frac{\partial u}{\partial m}\frac{\partial m}{\partial T}.$$

Households' disposable income $y = wl + T$ is increasing in the transfer, $\frac{\partial y}{\partial T} = 1$. Then, an increase in B induces a rise in T and therefore in y, and since both x and m are normal goods, $\frac{\partial x}{\partial T} > 0$ and $\frac{\partial m}{\partial T} > 0$. Therefore, $\frac{\partial V^c}{\partial B} > 0$. ■

Proposition 2 *For any given B, a lower realization of terms of trade P reduces household consumption $C(P,T)$ (and welfare) if there is no government intervention in asset markets, $T = 0$: If $P_1 < P_2$ then $u(C(P_1,0)) < u(C(P_2,0))$.*

Consider a decline in P. It must be proved that $\frac{dC(P,0)}{dP} > 0$.

In order to prove Proposition 2, Lemma 2.1 is proved.

Lemma 2.1 Equilibrium wages and inputs are increasing in terms of trade shocks: $\frac{dw(P)}{dP}, \frac{dL(P)}{dP}, \frac{dZ(P)}{dP} > 0$.

Proof. Given a GHH specification for households' preferences, labor decisions do not depend on the level of foreign assets. Therefore, equilibrium wages and inputs only depend on terms of trade: $w(P), L(P), Z(P)$.

Combining the optimal labor choice for the households $l = [\alpha^\alpha(1-\alpha)^{1-\alpha}\frac{w}{P^\alpha}]$ and the first order condition for firms $P\frac{\partial F(L,Z)}{\partial L} = w$, $P\frac{\partial F(L,Z)}{\partial Z} = 1$ one gets a system of two equations

$$\begin{aligned} P\frac{\partial F(L,Z)}{\partial L} &= P^\alpha G(L), \\ P\frac{\partial F(L,Z)}{\partial Z} &= 1 \end{aligned}$$

in two unknowns L and Z, where $G(L) = \frac{L^\Psi}{\alpha^\alpha(1-\alpha)^{1-\alpha}}$. Note that $\frac{\partial G(L)}{\partial L} > 0$ since $\Psi > 0$.

These two equations can be rewritten as:.

$$\begin{aligned} H_1(L(P), Z(P), P) &= 0, \\ H_2(L(P), Z(P), P) &= 0. \end{aligned}$$

Using Crammer's rule one can show that $\frac{dL(P)}{dP} > 0, \frac{dZ(P)}{dP} > 0:$

$$\begin{aligned} dH_1(\cdot) &= \frac{\partial H_1}{\partial L}\frac{dL(P)}{dP} + \frac{\partial H_1}{\partial Z}\frac{dZ(P)}{dP} + \frac{\partial H_1}{\partial P} = 0, \\ dH_2(\cdot) &= \frac{\partial H_2}{\partial L}\frac{dL(P)}{dP} + \frac{\partial H_2}{\partial Z}\frac{dZ(P)}{dP} + \frac{\partial H_2}{\partial P} = 0. \end{aligned}$$

A necessary and sufficient condition to solve for $\frac{dL(P)}{dP}$ and $\frac{dZ(P)}{dP}$ is that.

$$J = \frac{\partial H_1}{\partial L}\frac{\partial H_2}{\partial Z} + \frac{\partial H_1}{\partial Z}\frac{\partial H_2}{\partial L} \neq 0.$$

The elements of J have the following expressions:

$$\begin{aligned} \frac{\partial H_1}{\partial L} &= P\frac{\partial^2 F(L,Z)}{\partial L^2} - P^{\alpha}\frac{\partial G(L)}{\partial L} < 0, \\ \frac{\partial H_1}{\partial Z} &= P\frac{\partial^2 F(L,Z)}{\partial L \partial Z} > 0, \\ \frac{\partial H_2}{\partial L} &= P\frac{\partial^2 F(L,Z)}{\partial Z \partial L} > 0, \\ \frac{\partial H_2}{\partial Z} &= P\frac{\partial^2 F(L,Z)}{\partial Z^2} < 0 \end{aligned}$$

such that.

$$J = [P\frac{\partial^2 F(L,Z)}{\partial L^2} - P^{\alpha}\frac{\partial G(L)}{\partial L}]P\frac{\partial^2 F(L,Z)}{\partial Z^2} - P\frac{\partial^2 F(L,Z)}{\partial L \partial Z}P\frac{\partial^2 F(L,Z)}{\partial Z \partial L}.$$

Since

$$\frac{\partial^2 F(L,Z)}{\partial L^2}\frac{\partial^2 F(L,Z)}{\partial Z^2} - \left(\frac{\partial^2 F(L,Z)}{\partial L \partial Z}\right)^2 = 0.$$

then

$$J = -P^{1+\alpha}\frac{\partial G(L)}{\partial L}\frac{\partial^2 F(L,Z)}{\partial Z^2} \neq 0.$$

It follows that

$$\begin{aligned}
\frac{dL(P)}{dP} &= \frac{-\frac{\partial H_1}{\partial P}\frac{\partial H_2}{\partial Z} + \frac{\partial H_2}{\partial P}\frac{\partial H_1}{\partial Z}}{J}, \\
\frac{dZ(P)}{dP} &= \frac{-\frac{\partial H_1}{\partial L}\frac{\partial H_2}{\partial P} + \frac{\partial H_1}{\partial P}\frac{\partial H_2}{\partial L}}{J},
\end{aligned}$$

where.

$$\frac{\partial H_2}{\partial P} = \frac{\partial F(L,Z)}{\partial Z} > 0.$$

and since $\alpha \in (0,1)$ it follows from

$$H_1(L(P), Z(P), P) \equiv P\frac{\partial F(L,Z)}{\partial L} - P^\alpha G(L) = 0.$$

that

$$\frac{\partial H_1}{\partial P} = \frac{\partial F(L,Z)}{\partial L} - \alpha P^{\alpha-1}G(L) > 0.$$

then

$$\begin{aligned}
\frac{dL(P)}{dP} &= \frac{[\frac{\partial F(L,Z)}{\partial L} - \alpha P^{\alpha-1}G(L)]\frac{\partial^2 F(L,Z)}{\partial Z^2} - \frac{\partial F(L,Z)}{\partial Z}\frac{\partial^2 F(L,Z)}{\partial L \partial Z}}{P^\alpha \frac{\partial G(L)}{\partial L}\frac{\partial^2 F(L,Z)}{\partial Z^2}} > 0, \\
\frac{dZ(P)}{dP} &= \frac{[\frac{\partial^2 F(L,Z)}{\partial L^2} - P^{\alpha-1}\frac{\partial G(L)}{\partial L}] - [\frac{\partial F(L,Z)}{\partial L} - \alpha P^{\alpha-1}G(L)]\frac{\partial^2 F(L,Z)}{\partial Z \partial L}}{P^\alpha \frac{\partial G(L)}{\partial L}\frac{\partial^2 F(L,Z)}{\partial Z^2}} > 0.
\end{aligned}$$

Therefore, given the equilibrium wage $w = P^\alpha G(L)$ and $\frac{\partial G(L)}{\partial L} > 0$ and $\frac{dL(P)}{dP} > 0$.

$$\frac{dw(\cdot)}{dP} = P^{\alpha}\frac{\partial G(L)}{\partial L}\frac{dL(P)}{dP} + \alpha G(L)P^{\alpha-1} > 0.$$

These results are now used to prove Proposition 2, $\frac{dC(P,0)}{dP} > 0$.

$$\begin{aligned}
C(P,0) &= X(P,0)^{\alpha}M(P,0)^{1-\alpha} - \frac{L(P)^{1+\Psi}}{1+\Psi}, \\
X(P,0) &= \frac{\alpha}{P}w(P)L(P), \\
M(P,0) &= (1-\alpha)w(P)L(P), \\
L(P)^{\Psi} &= \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\frac{w(P)}{P^{\alpha}}.
\end{aligned}$$

Then:

$$\begin{aligned}
\frac{dC(P,0)}{dP} &= \alpha\left[\frac{M(P,0)}{X(P,0)}\right]^{1-\alpha}\left[\frac{\alpha}{P}\left(w(P)\frac{dL(P)}{dP} + L(P)\frac{dw(P)}{dP}\right) - \frac{\alpha}{P^2}w(P)L(P)\right] + \\
&\quad (1-\alpha)^2\left[\frac{X(P,0)}{M(P,0)}\right]^{\alpha}\left[w\frac{dL(P)}{dP} + L(P)\frac{dw(P)}{dP}\right] - L(P)^{\Psi}\frac{dL(P)}{dP}
\end{aligned}$$

and after substituting for X, M and L:

$$\frac{dC(P,0)}{dP} = \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\left[\frac{L(P)}{P^{\alpha}}\frac{dw(P)}{dP} - \alpha\frac{w(P)L(P)}{P^{1+\alpha}}\right].$$

Since

$$F(Z,L) = Z^{\gamma}L^{1-\gamma}$$

and

$$\frac{dw}{dP} = \frac{\partial F(L,Z)}{\partial L} - \frac{\frac{\partial F(L,Z)}{\partial Z}\frac{\partial^2 F(L,Z)}{\partial L\partial Z}}{\frac{\partial^2 F(L,Z)}{\partial Z\partial Z}}$$

it follows that:

$$\frac{dw}{dP} = Z^{\gamma}L^{-\gamma}.$$

Therefore, after substituting $\frac{dw}{dP}$ in $\frac{dC(P,0)}{dP}$:

$$\frac{dC(P,0)}{dP} = \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}[\frac{Z^{\gamma}L^{1-\gamma}}{P^{\alpha}} - \alpha wL]$$

and since $wL = \gamma PF(L,Z)$, $\alpha \in (0,1)$ and $\gamma \in (0,1)$, after some algebra the desired result is obtained:

$$\frac{dC(P,0)}{dP} = \frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{P^{\alpha}}[1-\alpha\gamma]F(L,Z) > 0.$$

■

Proposition 3 *For any given level of foreign debt, default incentives are stronger the lower the terms of trade. For all* $P_1 < P_2$*, if* $P_2 \in \digamma(B)$ *then* $P_1 \in \digamma(B)$.

In order to prove Proposition 3, we prove Lemma 3.1.

Lemma 3.1 If for some B *the default set is not empty* $\digamma(B) \neq 0$*, then there are no financial contracts available for the government* $\{q(B'), B'\}$ *such that the government could choose a positive transfer to the household* $T = B - q(B')B' > 0$.

Proof. This can be proven by contradiction. Suppose there are contracts available to the government such that $B - q(B')B' > 0$ but the government chooses B^* and $T^* = B - q(B^*)B^* < 0$ to maximize $V^c(B,P)$. Suppose as well that the government finds default to be optimal because $V^d(P) > V^c(B,P)$. Then,

$$\begin{aligned}
V^d(P) &= u(C(P,0)) + \beta EV^d(P'), \\
V^c(B,P) &= u(C(P,T^*)) + \beta EV_0(B^*,P'), \\
V^d(P) &> V^c(B,P), \\
u(C(P,0)) + \beta EV^d(P') &> u(C(P,T^*)) + \beta EV_0(B^*,P').
\end{aligned}$$

where

$$\begin{aligned}
C(P,T) &= X(P,T)^{\alpha} M(P,T)^{1-\alpha} - \frac{L(P)^{1+\Psi}}{1+\Psi}, \\
X(P,T) &= \frac{\alpha}{P}\left[w(P)L(P)+T\right], \\
M(P,T) &= (1-\alpha)\left[w(P)L(P)+T\right], \\
L(P) &= \left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\frac{W(P)}{P^{\alpha}}\right]^{\frac{1}{\Psi}}.
\end{aligned}$$

Note that without output loss in autarky $C^d(P) = C(P,0)$ since $w(P) = w^d(P)$ and $L(P) = L^d(P)$. Note that if there are financial contracts $\{q(B'), B'\}$ available to the government such that $B - q(B')B' > 0$ then default can not be the optimal decision, because it is possible for the government to choose a contract that implies a positive transfer $T > 0$ such that $V^c(B,P) > V^d(P)$, since $EV_0(B',P) \geqslant EV^d(P')$ and $u(C(P,T)) > u(C(P,0))$. The first expression comes from the fact that $V_0(B',P') = \max\left\{V^c(B',P'), V^d(P')\right\}$ and the second expression holds because the transfer is positive and $C(P,T)$ is increasing in T since both goods are normal.

$$\begin{aligned}
\frac{dC(P,T)}{dT} &= \alpha\left[\frac{M(P,T)}{X(P,T)}\right]^{(1-\alpha)}\frac{dX(P,T)}{dT} + (1-\alpha)\left[\frac{X(P,T)}{M(P,T)}\right]^{\alpha}\frac{dM(P,T)}{dT} > 0, \\
\frac{dX(P,T)}{dT} &= \frac{\alpha}{P} > 0, \\
\frac{dM(P,T)}{dT} &= (1-\alpha) > 0.
\end{aligned}$$

Thus, B^* can not be the level of foreign assets that maximizes $V^c(B,P)$ and then find default to be optimal. There are contracts available to the government such that the value of staying in the contract is higher than the value of defaulting and going to autarky. Thus , if $F(B) \neq 0$, then $\exists$ some P, such that $V^d(P) > V^c(B,P)$ therefore if B' is chosen to maximize $V^c(B,P)$ and default is the optimal decision, it must be the case that there are no financial contracts $\{q(B'), B'\}$ that allowed the government to give a positive transfer to the household, thus $T = B - q(B')B' < 0$ for all financial contracts $\{q(B'), B'\}$.

Now, we can prove Proposition 3, which says that for all $P_1 < P_2$, if $P_2 \in F(B)$ then $P_1 \in F(B)$. Now if $P_2 \in F(B)$ then $V^d(P_2) > V^c(B, P_2)$.

$$
\begin{aligned}
u(C(P_2, 0)) + \beta E V^d(P') &> u(C(P_2, T_2)) + \beta E V_0(B_2, P'), \\
\{T_2, B_2\} &\in \arg\max V^c(B, P_2), \\
T_2 &< 0.
\end{aligned}
$$

The transfer T_2 is negative by Lemma 3.1. If the following inequality 7.1 holds, then $P_2 \in F(B)$ implies $P_1 \in F(B)$.

$$
V_c(B, P_2) - V_c(B, P_1) > V^d(P_2) - V^d(P_1). \tag{7.1}
$$

Where

$$
\begin{aligned}
V_c(B, P_2) - V_c(B, P_1) &= u(C(P_2, T_2)) + \beta E V_0(B_2, P') - .. \\
&\quad .. - \{u(C(P_1, T_1)) + \beta E V_0(B_1, P')\} ..
\end{aligned}
$$

$$
\begin{aligned}
V^d(P_2) - V^d(P_1) &= u(C(P_2, 0)) + \beta E V^d(P') - .. \\
&\quad .. - \left\{u(C(P_1, 0)) + \beta E V^d(P')\right\}.
\end{aligned} \tag{7.2}
$$

Condition 7.1 implies that given foreign assets, default is also optimal when terms of trade are lower than P_2.

$$
V^d(P_1) - V_c(B, P_1) > V^d(P_2) - V_c(B, P_2) > 0.
$$

In order to prove Proposition 3 we prove that condition 7.1 holds. Given that terms of trade shock are i.i.d. the right hand side of 7.2 simplifies into:

$$V^d(P_2) - V^d(P_1) = u(C(P_2, 0)) - u(C(P_1, 0)).$$

Given B and P_2, $\{T_2, B_2\}$ are the optimal decisions for transfer and foreign assets, so it must be the case that:

$$u(C(P_2, T_2)) + \beta EV_0(B_2, P') \geqslant u(C(P_2, T_1)) + \beta EV_0(B_1, P').$$

If the following condition 7.3 holds then 7.1 holds by transitivity

$$\begin{aligned} u(C(P_2, 0)) - u(C(P_1, 0)) \quad < \quad & u(C(P_2, T_1)) + \beta EV_0(B_1, P') - .. \\ & .. - \left\{u(C(P_1, T_1)) + \beta EV_0(B_1, P')\right\}. \end{aligned} \tag{7.3}$$

We can simplify 7.3 to get 7.4:

$$u(C(P_2, T_1)) - u(C(P_1, T_1)) > u(C(P_2, 0)) - u(C(P_1, 0)). \tag{7.4}$$

Since $u(C)$ is increasing in C and concave, 7.4 holds if the following condition is satisfied:

$$C(P_2, T_1) - C(P_1, T_1) \geqslant C(P_2, 0) - C(P_1, 0). \tag{7.5}$$

Given $\frac{dC(P,T)}{dT} > 0$, 7.5 holds for $T_1 < 0$.

$$\begin{aligned} C(P_2, T_1) - C(P_1, T_1) &= X(P_2, T_1)^{\alpha} M(P_2, T_1)^{1-\alpha} - X(P_1, T_1)^{\alpha} M(P_1, T_1)^{1-\alpha}, \\ C(P_2, 0) - C(P_1, 0) &= X(P_2, 0)^{\alpha} M(P_2, 0)^{1-\alpha} - X(P_1, 0)^{\alpha} M(P_1, 0)^{1-\alpha}. \end{aligned}$$

After some algebra and substituting the equilibrium expressions for X and M, the following result is obtained

$$\begin{aligned} C(P_2, T_1) - C(P_1, T_1) &= \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\left[\frac{w(P_2)L(P_2) + T_1}{P_2^{\alpha}} - \frac{w(P_1)L(P_1) + T_1}{P_1^{\alpha}}\right], \\ C(P_2, 0) - C(P_1, 0) &= \alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\left[\frac{w(P_2)L(P_2)}{P_2^{\alpha}} - \frac{w(P_1)L(P_1)}{P_1^{\alpha}}\right]. \end{aligned}$$

Thus, since $P_1 < P_2$, 7.1 holds if $T_1 < 0$.

$$C(P_2, T_1) - C(P_1, T_1) - \{C(P_2, 0) - C(P_1, 0)\} = \alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \left[\frac{1}{P_2^{\alpha}} - \frac{1}{P_1^{\alpha}}\right] T_1 > 0.$$

Therefore, 7.1 holds which implies that $P_1 \in F(B)$. ■

Proposition 4 *Asset decisions are increasing in terms of trade. The government borrows more in recessions than in booms. Given a current level of foreign assets B and conditional on not defaulting, for all $P_1 < P_2$, if $B_1' = B'(B, P_1)$ and $B_2' = B'(B, P_2)$ then $B_1' < B_2'$.*

Proof. This proposition can be proven by contradiction. Suppose that $P_1 < P_2$ and $B_1' \geqslant B_2'$. The government, conditional on not defaulting, borrows more in good times than in bad times.

$$\begin{aligned} \{T_1, B_1\} &\in \arg\max V^c(B, P_1), \\ \{T_2, B_2\} &\in \arg\max V^c(B, P_2). \end{aligned}$$

From utility maximization:

$$u(C(P_1, T_1)) + \beta EV_0(B_1', P') \geqslant u(C(P_1, T_2)) + \beta EV_0(B_2', P'), \tag{7.6}$$

$$u(C(P_2, T_2)) + \beta EV_0(B_2', P') \geqslant u(C(P_2, T_1)) + \beta EV_0(B_1', P'). \tag{7.7}$$

If $B_1' \geqslant B_2'$. then $EV_0(B_1', P') \geqslant EV_0(B_2', P')$ because

$$V_0(B_i', P') = \max\left\{V^c(B_i', P'), V^d(P')\right\}.$$

and the value of no defaulting is strictly increasing in foreign assets. In addition, If $EV_0(B_1', P') \geqslant EV_0(B_2', P')$ then expression 7.7 implies that $C(P_2, T_2) \geqslant C(P_2, T_1)$. Since $C(P, T)$ is increasing in the transfer, T_2 would be higher than T_1.

$$T_2 = B - q(B_2') B_2' \geqslant B - q(B_1') B_1' = T_1.$$

Adding expressions 7.6 and 7.7, the following is derived:

$$
\begin{aligned}
u\left(C\left(P_2,T_2\right)\right)+u\left(C\left(P_1,T_1\right)\right) &\geqslant u\left(C\left(P_2,T_1\right)\right)+u\left(C\left(P_1,T_2\right)\right), \\
u\left(C\left(P_2,T_2\right)\right)-u\left(C\left(P_2,T_1\right)\right) &\geqslant u\left(C\left(P_1,T_2\right)\right)-u\left(C\left(P_1,T_1\right)\right).
\end{aligned}
$$

Since $\frac{\partial C(P,T)}{\partial P}>0$ by Proposition 2, and utility is concave and increasing in $C\left(P,T\right)$, the last expression does not hold if

$$
C\left(P_2,T_2\right)-C\left(P_2,T_1\right)\leqslant C\left(P_1,T_2\right)-C\left(P_1,T_1\right). \tag{7.8}
$$

In order to prove Proposition 4 it is sufficient to prove that the above equation 7.8 holds for all $P_1<P_2$ and $T_1\leq T_2$.

$$
\begin{aligned}
C\left(P_2,T_2\right)-C\left(P_2,T_1\right) &= X\left(P_2,T_2\right)^{\alpha}M\left(P_2,T_2\right)^{1-\alpha}-X\left(P_2,T_1\right)^{\alpha}M\left(P_2,T_1\right)^{1-\alpha}, \\
C\left(P_1,T_2\right)-C\left(P_1,T_1\right) &= X\left(P_1,T_2\right)^{\alpha}M\left(P_1,T_2\right)^{1-\alpha}-X\left(P_1,T_1\right)^{\alpha}M\left(P_1,T_1\right)^{1-\alpha}.
\end{aligned}
$$

After some algebra and substituting the equilibrium expressions for X and M, the following result is obtained

$$
\begin{aligned}
C\left(P_2,T_2\right)-C\left(P_2,T_1\right) &= \alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}\frac{1}{P_2^{\alpha}}\left[T_2-T_1\right], \\
C\left(P_1,T_2\right)-C\left(P_1,T_1\right) &= \alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}\frac{1}{P_1^{\alpha}}\left[T_2-T_1\right].
\end{aligned}
$$

Thus, since $P_1<P_2$, expression 7.8 holds if $T_1\leq T_2$.

$$
C\left(P_2,T_2\right)-C\left(P_2,T_1\right)-\left\{C\left(P_1,T_2\right)-C\left(P_1,T_1\right)\right\}=\alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}\left[\frac{1}{P_2^{\alpha}}-\frac{1}{P_1^{\alpha}}\right]\left[T_2-T_1\right]\leq 0.
$$

Therefore, condition 7.8 holds which implies that $B_1'\leqslant B_2'$. ■

Proposition 5 *Interest rates cannot be procyclical. Given the level of foreign assets* B *, for all* $P_1<P_2$*, if* $B_1'=B'\left(B,P_1\right)$ *and* $B_2'=B'\left(B,P_2\right)$ *then* $q\left(B_1'\right)\leqslant q\left(B_2'\right)$.

Proof. This proposition can be proven by contradiction. Suppose that $P_1 < P_2$ and $q(B_1') > q(B_2')$. Since shocks are assumed to be i.i.d., the bond price schedule does not depend on the current level of terms of trade. According to Lemma 2.1 $GDP(P_1) < GDP(P_2)$ since both inputs are increasing in terms of trade. Thus, bond prices would be lower in booms, which means that interests rates, which are equal to the inverse of bond prices, are procyclical. Since bond prices are decreasing in default probabilities, a higher bond price in recessions implies a procyclical default probability.

$$\begin{aligned} q\left(B_i'\right) &= \frac{(1-\lambda_i(B_i'))}{1+r_f}, \\ q\left(B_1'\right) &> q\left(B_2'\right), \\ \lambda_1(B_1') &< \lambda_2(B_2'). \end{aligned}$$

Default probabilities depend on default sets

$$\sum_{F\left(B_1'\right)} \pi\left(P\right) < \sum_{F\left(B_2'\right)} \pi\left(P\right).$$

According to Propositions 4 and 1, if $P_1 < P_2$ then $B_1' \leqslant B_2'$, and $F\left(B_2'\right) \subset F\left(B_1'\right)$, which means that the previous expression can not hold, thus interest rates can not be procyclical. ■

7.3. Tables and Figures

Table 1 Terms of Trade, 1994.1 - 2004.4

	Std Dev	Correlations				
		Argentina	Colombia	Ecuador	Peru	Venezuela
Argentina	4.22		0.18	0.55	0.47	0.39
Colombia	6.35	0.18		0.38	0.39	0.16
Ecuador	11.27	0.55	0.38		0.31	0.85
Peru	8.70	0.47	0.39	0.31		0.21
Venezuela	24.93	0.39	0.16	0.85	0.21	

Table 2 Emerging Economies Data, 1994.1 - 2004.4

	Std Dev(TOT)	Std Dev(GDP)	Std Dev(Spread)	Corr(GDP-Spread)
Argentina	4.22	4.98	10.17	-0.65
Colombia	6.35	2.05	1.15	0.02
Ecuador	11.27	3.01	6.92	-0.70
Peru	8.70	2.32	1.06	-0.47
Venezuela	24.93	6.22	2.45	-0.55

Table 3 Cross Correlations, 1994.1 - 2004.4

	Correlation of GDP(t) with				
	TOT(t)	TOT(t-1)	TOT(t-2)	TOT(t-3)	TOT(t-4)
Argentina	0.20	0.31	0.33	0.32	0.20
Colombia	0.17	0.30	0.33	0.25	0.11
Ecuador	-0.27	-0.06	0.23	0.51	0.72
Peru	0.53	0.59	0.40	0.18	0.01
Venezuela	0.01	0.07	0.08	0.47	0.55

Table 4 Benchmark Parameter Values

Discount Factor	β	0.85
Risk Aversion	σ	2
Domestic Good Share	α	0.85
Elasticity of Labor Supply	$\frac{1}{\Psi}$	2.22
Re-entry Probability	μ	0.1
Foreign Input Share	$1-\theta$	0.39
Terms of Trade	σ_{ε^p}	0.079
	ρ_p	0.68
U.S. Real Interest Rate	r_f	0.01
Factor Productivity in Autarky	A	0.98

Table 5 Simulation Results			
	Std Dev	Correlation with GDP	Correlation with Spread
GDP	5.10		-0.38
Consumption	4.98	0.98	-0.36
Terms of Trade	10.89	1	-0.38
Spread	0.69	-0.38	
Default Rate	0.0032		
Risk Free debt Limit	16.07		
Maximum Spread	2.25		

Table 6 Sensitivity Analysis					
	Std Dev Spread	Maximum Spread	Correlation (GDP-Spread)	Default Rate	Risk Free Debt Limit
Input Share	(Production)				
0.2	0.31	0.99	-0.25	0.0010	0.2014
0.39	0.69	2.25	-0.38	0.0032	0.1607
0.5	0.98	5.94	-0.42	0.0047	0.1222
Domestic Good Share	(Utility)				
0.95	0.52	1.47	-0.27	0.0019	0.1703
0.85	0.69	2.25	-0.38	0.0032	0.1607
0.75	0.81	2.37	-0.36	0.0035	0.1496

Figure 1
GDP-TOT Ecuador, 1995-2004

Figure 2
GDP-TOT Venezuela, 1995-2004

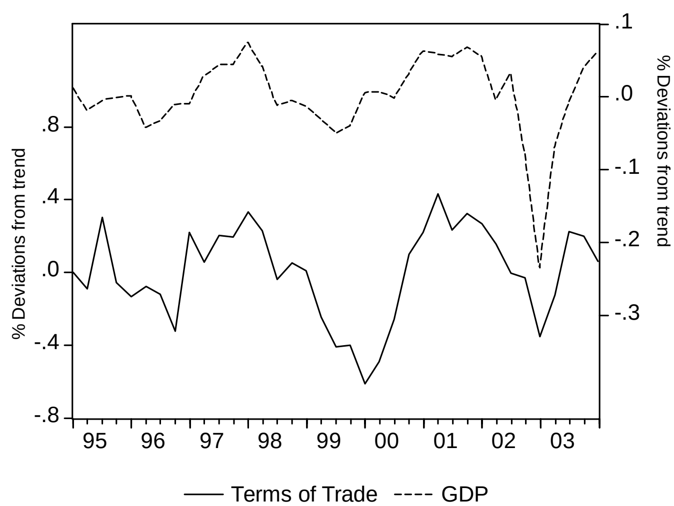


Figure 3
GDP-Spread Ecuador, 1995-2004

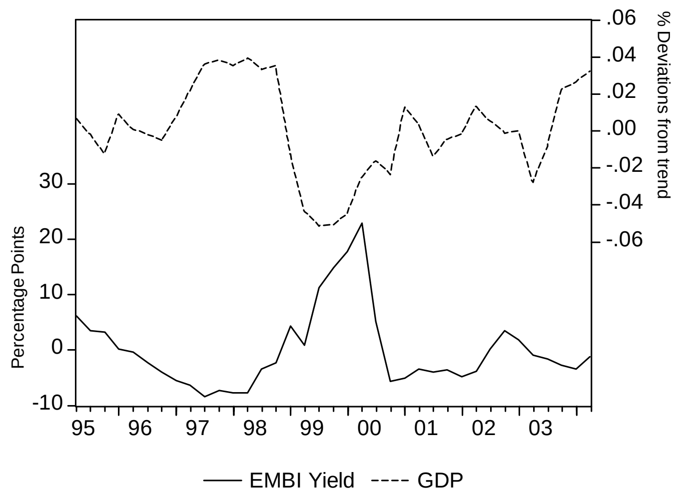


Figure 4
GDP-Spread Venezuela, 1998-2004

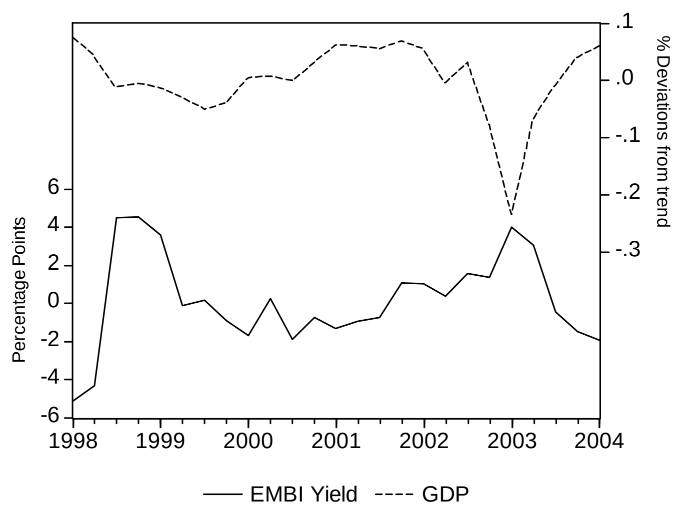


Figure 5
Terms of Trade, 1994-2004

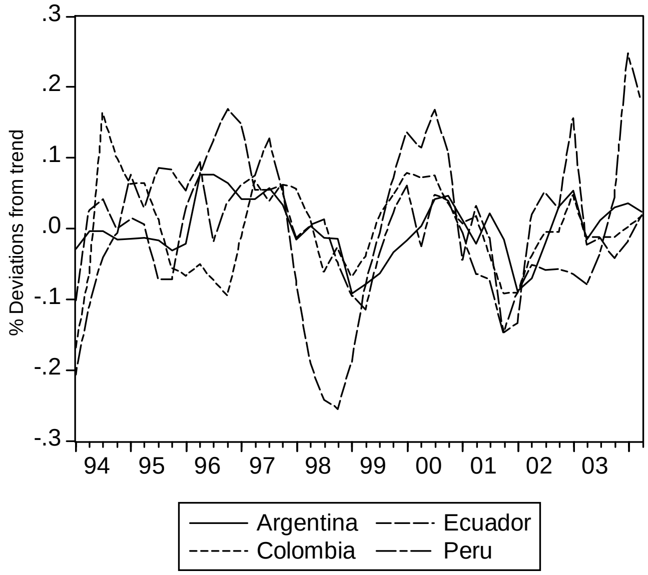


Figure 6. Bond Price

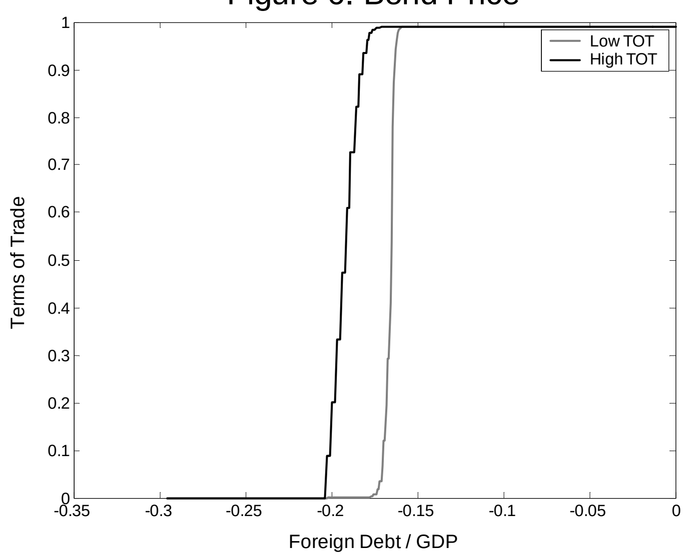


Figure 7. Default Region

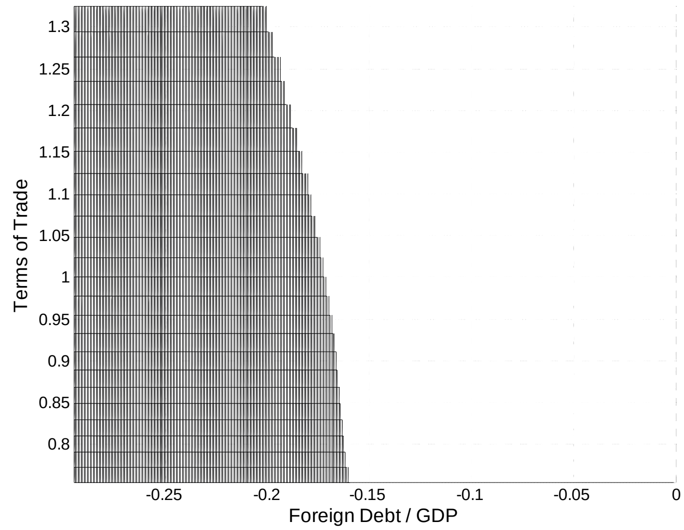


Figure 8
Dynamics Prior to Default

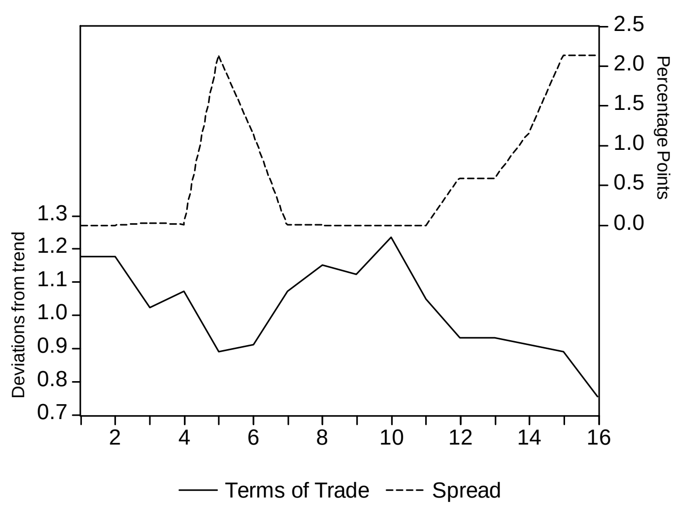
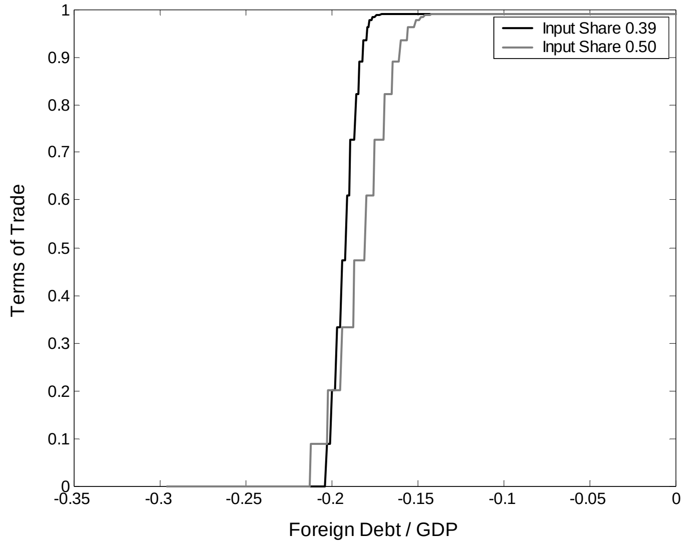


Figure 9. Bond Price and Input Share



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